Model Answer

Mechanics-I (AV-8880)

B. Sc. (5^{th} Semester) Examination 2015-16

1.(i) Common Catenary: The curve in which a uniform chain or string (perfectly flexible) hangs freely suspended from two fixed points is called common catenary.

$$\boxed{s = c\tan\psi}$$

is called intrinsic equation of common catenary.

(ii) We know that

$$s = c\tan\psi$$

$$\Rightarrow \quad \frac{ds}{d\psi} = c\sec^2\psi$$

$$\Rightarrow \quad \frac{dy}{d\psi} = \frac{dy}{ds}\cdot\frac{ds}{d\psi}$$

$$= \sin\psi\cdot c\sec^2\psi$$

Since $\frac{dy}{ds} = \sin\psi$

$$\Rightarrow \quad \frac{dy}{d\psi} = c\tan\psi\sec\psi$$

$$\Rightarrow \quad y = c\sec\psi + C$$

For catenary $y = c,\ \psi = 0$

$$\Rightarrow \quad C = 0$$

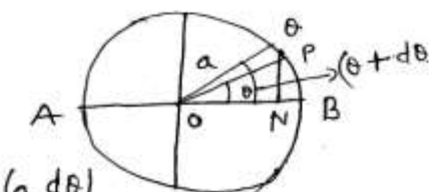
Hence $\boxed{y = \sec\psi}$

(iii) Product of inertia: $\int xy\,dm$ is defined as product of inertia of lamina with respect to the axes OX and OY.

(iv) Energy test for Stability: We know that stability depends ~~on~~ upon the height of the center of gravity. ~~If~~ Hence for maximum height of the cener of gravity we have unstable equillibrium in this cas we have maximum potential energy and for minimum hight of the center of gravity we ~~have~~ have stable equilibrium in this case we have minimum potential energy.

(V). Moment of inertia of a circular wire about diameter:

Let M be the mass amd 'a' be the radius of the circular wire.



mass per unit length $= \dfrac{M}{2\pi a}$

mass of elemetry part $PQ = \dfrac{M}{2\pi a}(a\,d\theta)$

moment of inertia of the circular wire about AB

$$= \int_0^{\pi/2} \left(\frac{M}{2\pi a}\right)(a\,d\theta)\, a^2\sin^2\theta$$

$$= \frac{Ma^2}{2}.$$

(vi) D'Alembert's principle: The principle states that the sum of the differences between the forces acting on a system of mass particles and the time derivatives of the momenta of the system itself along any virtual displacement consistent with constraints of the system is zero. i.e

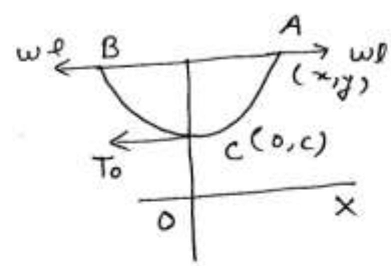
$$\sum_i (F_i - m_i a_i) \cdot \delta r_i = 0.$$

(vii) Equimomental system: Two mechanical systems are said to be equimomental if there moments of inertia about all lines are same.

(viii) Stable Equilibrium: If a body is slightly displaced from its original position of equilibrium the forces acting on it may be such that they may bring the body back to its position of equilibrium is said to be stable.

2 Let string rest in the form of a catenary ACB. Let its equation be

$$s = c \tan \psi \quad \text{—— (1)}$$



Let coordinate of A be (x, y)

For equilibrium

$$F = wl = T_0 = wc$$

$$\Rightarrow c = l$$

Again for the point A, $s = l$

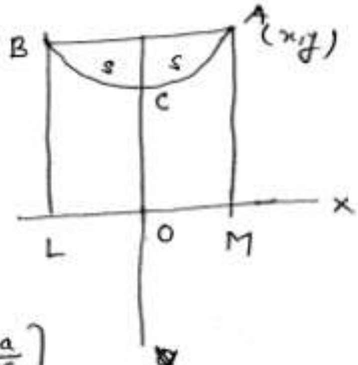
$$l = c \tan\psi$$
$$\Rightarrow \quad l = l \tan\psi$$
$$\Rightarrow \quad \tan\psi = 1$$
$$\psi = 45^0$$

Now the distance between A and B (between the two rings)

$$AB = 2x = 2c \log(\sec\psi + \tan\psi)$$
$$= 2l \log(\sec 45^0 + \tan 45^0)$$
$$= 2l \log(1 + \sqrt{2}).$$

Ans.

3 Let LBCAM be the heavy string hanging ove two fixed pegs at A and B.



The total length of the string

$$l = 2(y + s)$$
$$= 2\left[c \cosh\frac{a}{c} + c \sinh\frac{a}{c}\right]$$

$$l = 2c\, e^{a/c}$$

Length will be least if

$$\frac{dl}{dc} = 0$$

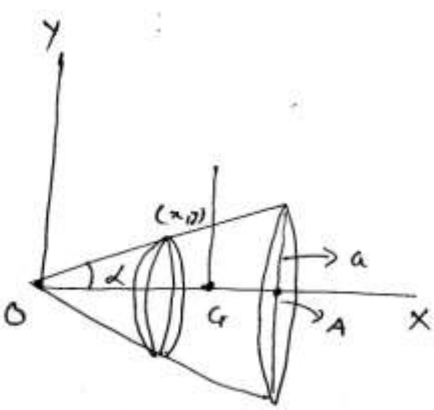
$$\Rightarrow \quad 2e^{a/c} - 2c\, e^{a/c} \cdot \frac{a}{c^2} = 0$$

$$\Rightarrow \quad c = a$$

Hence $l = 2a\, e^{a/a} = 2ae$. This is least because $\frac{d^2l}{dc^2} > 0$.

Ans.

4 Moment of inertia of solid ~~hollow~~ Cone.



From fig -

$$y = x \tan\alpha \quad \text{---} \quad (I)$$

$$\tan\alpha = \frac{a}{h} \quad \text{---} \quad (II)$$

$OG = \frac{3}{4} h$

$OA = h$

$\rho \equiv$ density.

Mass of elementry part

$$= \left(\pi x^2 \tan^2\alpha \, dx\right)\rho$$

Moment of inertia of solid cone about Oy

$$= \int_0^h \pi x^2 \tan^2\alpha \, \rho \left(\frac{x^2 \tan^2\alpha}{4} + x^2 \right)$$

$$= \frac{3M}{20} \left(a^2 + 4h^2 \right)$$

Now the moment of inertia about a line through G and $\perp$ar to the axis $= \frac{3M}{20}\left(a^2 + 4h^2\right) - M \cdot \frac{9h^2}{16}$

$$= \frac{3M}{80} \left(h^2 + 4a^2 \right)$$

Note: Similarly we can find the moment of inertia of hollow cone.

5 Theorem of parallel axis: If the moments and products of inertia about any axis through the centre of gravity of a body are known, to find the moments and products of inertia about a parallel axis.

Let $G(\bar{x}, \bar{y}, \bar{z})$ be the centre of gravity of the body referred to ~~rag~~ rectangular axes OX, OY and OZ. Let (x, y, z) be the coordinates of a point P referred to these axes and (x', y', z') be the coordinates of the same point referred to parallel axes Gx', Gy', Gz'

so that

$$x = \bar{x} + x', \quad y = y' + \bar{y}, \quad z = z' + \bar{z}$$

If m be the mass at P then moment of inertia I of the body about Ox is given by

$$I = \Sigma m (y^2 + z^2)$$

$$= \Sigma m \left[(y' + \bar{y})^2 + (z' + \bar{z})^2\right]$$

But $\Sigma m y' = 0$, $\Sigma m z' = 0$.

Hence $I = \Sigma m (y'^2 + z'^2) + m(\bar{y}^2 + \bar{z}^2)$

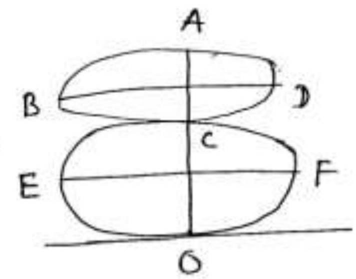
= Moment of inertia about Gx' + mass × (distance of centre of gravity from ox)

$$\boxed{I = I_c + Md^2}$$

Similarly we can show for product of inertia.

6

Let ABCD and CEOF be the cross sections of the two cylinders and AC, CO, their minor axes.



If the equation to the either ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ∴

The height of the C.G. of the system, above O is $2b$.

therefore for stable equilibrium

$$\frac{1}{2b} > \frac{b}{a^2}+\frac{1}{\infty}$$

$$\Rightarrow \quad a^2 > 2b^2$$

$$\Rightarrow \quad a^2 > 2a^2(1-e^2)$$

$$\Rightarrow \quad 1 > (2-2e^2)$$

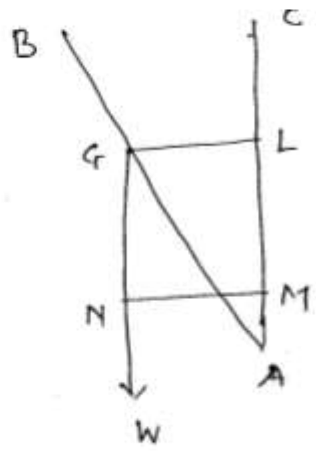
$$\Rightarrow \quad 2e^2 > 1$$

$$\Rightarrow \quad e > \frac{1}{\sqrt{2}}.$$

Ans.

7.

Let $2a$ be the length of the rod AB.



let z be the height of the centre of gravity

$$z = GN = LM$$
$$= LA - AM$$
$$= AG\cos\theta - PM\cos\theta$$
$$= a\cos\theta - b\cot\theta$$

$$\frac{dz}{d\theta} = -a\sin\theta + b\,\mathrm{cosec}^2\theta$$

For equilibrium $\frac{dz}{d\theta} = 0$

$$\therefore a\sin^2\theta = b\,\mathrm{cosec}^2\theta$$

$$\Rightarrow \sin^3\theta = b/a$$

This gives the position of equilibrium

$$\frac{d^2z}{d\theta^2} = -\cos\theta\,(a + 2b\,\mathrm{cosec}^3\theta)$$

$\therefore$ when $\sin^3\theta = b/a$

$$\Rightarrow \frac{d^2z}{d\theta^2} = -3a^{2/3}(a^{2/3} - b^{2/3})$$

z is max.

$\Rightarrow$ The equilibrium is unstable.

8- Motion of a compound pendulum: A compound pendulum is one where the rod is not massless, and may have extended size, that is, an arbitrarily shaped rigid body swinging by a pivot. In case pendulum periods depends on its moment of inertia I around the pivot point.

The equation of torque is given by $\tau = I\alpha$, —(I)

where α is the angular acceleration, τ is the torque.

Since torque is generated by gravity so

$$\tau = -mgl \sin\theta \qquad \text{——— (II)}$$

For small angle approx. $\sin\theta \cong \theta$

From (II) & (I), we have

$$\alpha \simeq \frac{mgl\,\theta}{I}$$

This gives a period of

$$\boxed{\tau = 2\pi\sqrt{\frac{I}{mgl}}}$$

Ans